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MODELS FOR CALCULATING MULTIPLE ROUND
HIT PROBABILITY WITH 4 BOMBS

by

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Thesis Advisor: Alan R. Washburn

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Models for calculating multiple round hit probability (the chance of at least one hit) with 4 bombs on a circular target are constructed by computer simulation. Pattern firing and artillery registration are compared to determine which firing method is optimal. It is proved neither one is optimal. Therefore, another method called modified artillery registration is developed. The basic idea of modified artillery registration is feed back superimposed on a pattern. This method always gives higher probability than the former two methods.

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I. INTRODUCTION

A frequent and important problem facing the weapon systems analyst is that of making calculations of hit probabilities for multiple rounds, and finding the best method to increase the hit probability with the given rounds. The purpose of this thesis is to present models for calculation of the probability of at least one hit, and to find the firing method that maximizes the hit probability under the following assumptions:

A. 4 bombs are fired at a circular target of radius R whose position is drawn from a circular normal distribution with mean at the origin of Cartesian coordinate and variance σ_1^2. This is called "target location" error.

B. The "round-to-round" errors are governed by a circular normal distribution with mean 0 and variance σ_2^2 .

C. In cases where an observer is involved, the "report" errors are also distributed by a circular normal distribution with mean 0 and variance σ_3^2.

D. Even though observers in real life tend to make larger errors when observing large miss distance, the three errors above, referred to as the "target location" error, "round-to-round" error and "report" error, respectively, are assumed to be independent.

A typical multiple rounds firing is shown in Figure 1 for N = 4 where the circular dot represents the target position with radius R (the center is drawn from a circular normal

distribution with mean 0 and variance σ_1^2) and the crosses represent the individual round impact points (drawn independently from a circular normal distribution centered on the origin with mean 0 and variance σ_2^2). If any one of four bombs hits within the target radius, the target is considered to be killed.

The rest of this thesis will examine several different methods for aiming the 4 shots.

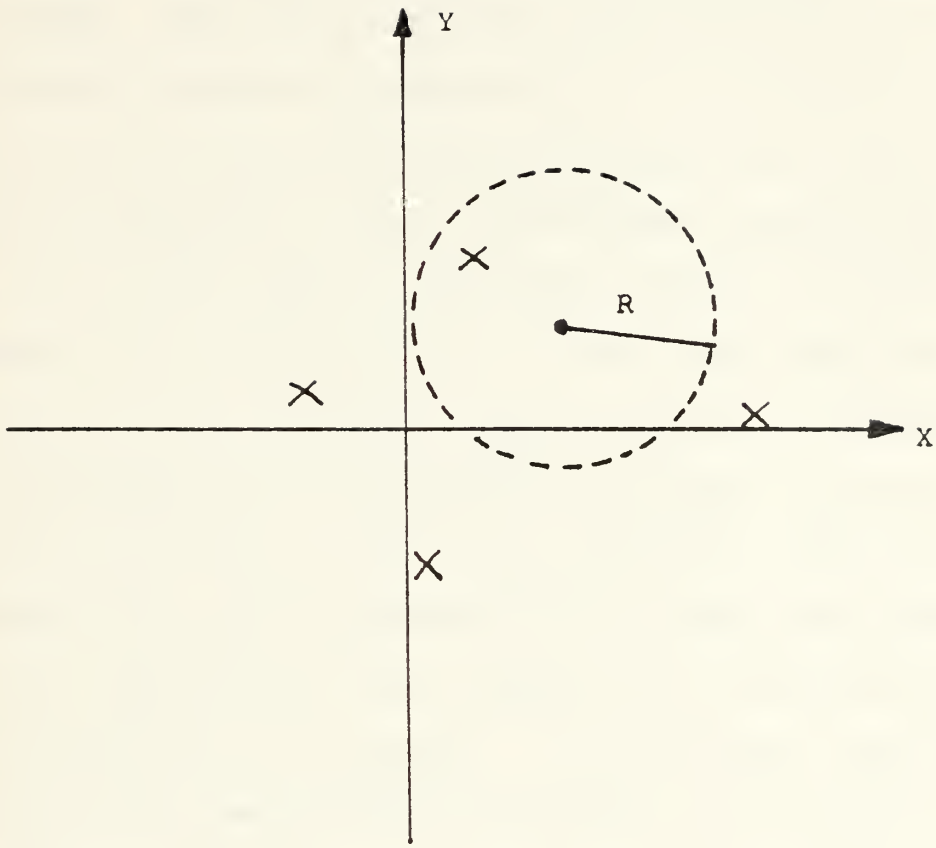

Figure 1: Typical Multiple Rounds Pattern for N = 4

II. DISCUSSION OF PATTERN FIRING METHOD

As discussed in the previous chapter, a target is considered to be destroyed if any one of 4 bombs lies within a lethal radius of the target. The hit probability formulation can be obtained by writing the conditional probability of at least one hit in density form and using the classical probability rules pertaining to conditional, joint, and marginal probability densities. For a given set of aim points, the hit probability (probability of at least one hit) is then some complicated function of σ_1, σ_2, σ_3 and the lethal radius R. In general, the hit probability is not maximized by aiming all shots at the origin but rather in some sort of pattern. Naturally, one expects that the optimal pattern will be some simple geometric figure. For three rounds, for example, the aim point pattern should be vertices of a triangle. In this thesis, the four aim points will be at the corners of a square, since there is some evidence developed by Washburn that this is superior to the "triangle-plus-one-in-the-middle" pattern.

Figure 2 shows the pattern firing method. Target location is a random number from circular normal distribution with mean 0 and variance σ_1^2. The impact point is the aiming point plus "round-to-round" error which is drawn from a normal distribution with mean 0 and variance σ_2^2. In Figure 2, the triangle represents the target location, the crosses 1,2,3,4 are the aiming points of four bombs which are the corners of a square, and the

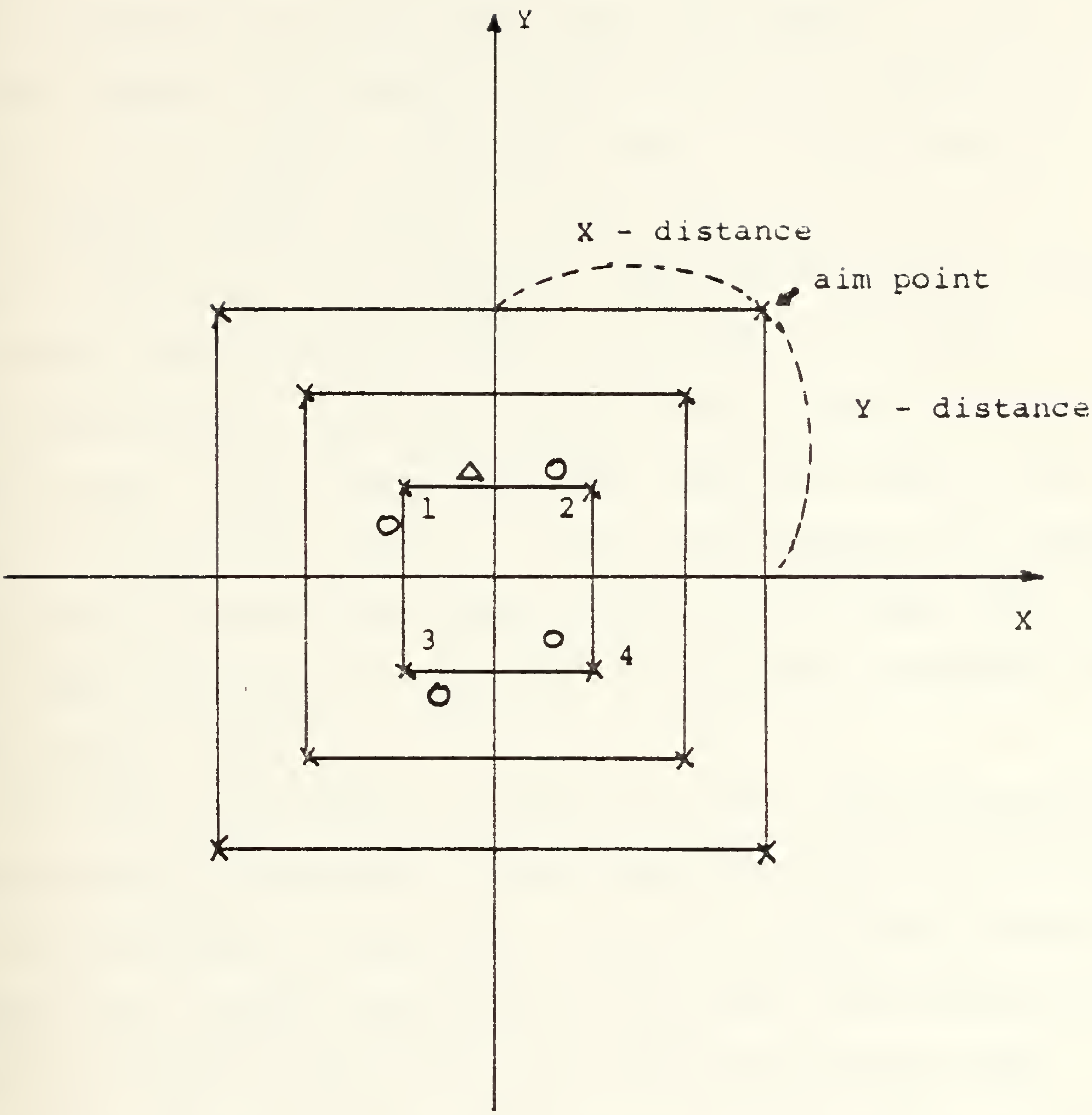

Figure 2: Pattern Firing Method

4 circular dots represent the real impact points. Therefore, if one shoots all 4 bombs aimed at the center of Cartesian coordinates, the real impact points are the "round-to-round" error points. So the target is considered to be destroyed if the target location is in the lethal area with radius R of any one of the 4 bombs. For example, suppose the target coordinates are (-2, 5), and that 4 bombs with lethal radius 2 are shot with "round-to-round" errors (-1,2), (2,1), (2,-3), (-2,-2). Figure 3 represents the situation. The dotted circles represent the lethal region when all shots are aimed at the center. As the picture shows, the target doesn't lie in any one of the lethal areas. In the case where the shots are aimed at the points (-2,2), (2,2), (2,-2), (-2,-2), the corners of a square, the crosses represent the impact points of the bomb. The picture shows the target (-2,5) lies in the circle associated with impact point (-3,4).

The above illustration presents the case that the pattern firing method can be better than firing all shots at the center.

In the computer simulation, 10,000 X-coordinates are generated from a normal distribution with mean 0 and variance σ_1^2 and 10,000 Y-coordinates are also generated from the same normal distribution as target locations. For each target location, 4 round-to-round error X-coordinates and 4 Y-coordinates are generated from a normal distribution with mean 0 and variance σ_2^2 . The square pattern size is gradually increased to determine the best pattern size. Table I and Table II illustrate the computer run results. In the table, the upper right

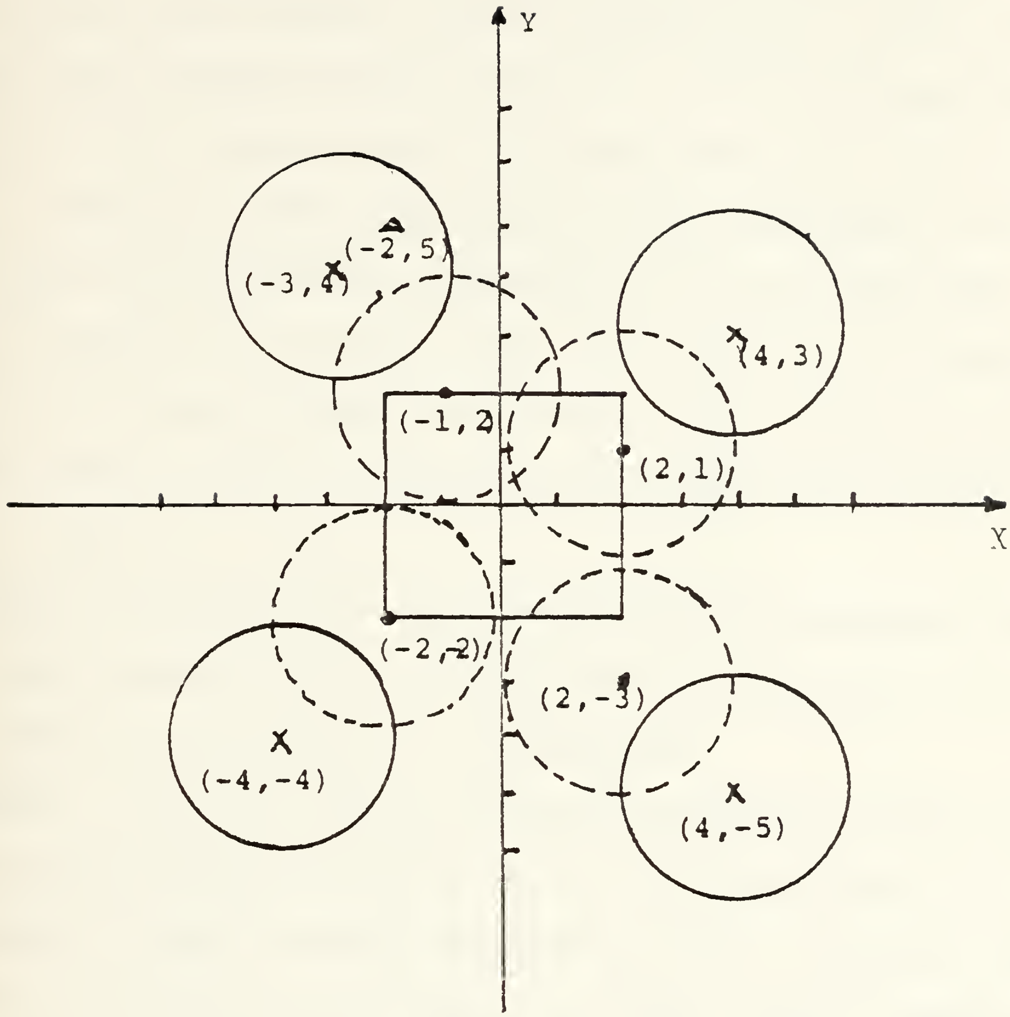

Figure 3: An Example of Hit and No-Hit with Pattern Firing

aim point (X = Y) 3 represents the first corner (3,3) of the square. Therefore, the other three aiming points should be (3,-3), (-3,3), and (-3,-3). Table I shows that the pattern firing method is better than aiming at the center. Because when aimed at the center, the hit probability is 0.611 and hit probability increases as the size of a square increases and reaches at the maximum value 0.651 when one aims (2,2), 2,-2), (-2,2), (-2,-2). After that point if one increases the pattern size, the probability decreases. On the other hand, Table II shows a case where there is no necessity to use a pattern. If one increases the pattern size instead of aiming at the center, the hit probability decreases from 0.430 to 0.427, 0.404, . . . etc.

Table III illlustrates the results of the computer simulation with lethal radius 5, and target location error (σ_1) 5, with round-to-round error (σ_2) variables.

When σ_2 is 0.05, the hit probability aiming at the center is 0.527 and the best pattern firing hit probability is 0.780. As "round-to-round" error increases, pattern firing hit probability decreases and the size of the best square becomes smaller and smaller. But the hit probability aiming at the center increases to the value 0.608 when σ_2 is 4. When σ_2 is larger than 4, all the fires should be aimed at the center. That means intentional dispersion decreases hit probability when "round-to-round" error is larger than 4.

Another important fact is that the size of the pattern should become larger as the lethal radius becomes larger. For example,

Lethal Radius = 5.0

Target Location Error = Normal $(0,5^2)$

Round-to-Round Error = Normal $(0,3^2)$

Aim Point (X = Y)	hit probability
0	0.611
1	0.638
2	0.651
3	0.631
4	0.565
5	0.468
6	0.364
7	0.259
8	0.171
9	0.109
10	0.068
11	0.036
12	0.020
13	0.011
14	0.004
15	0.002

TABLE II: Pattern Firing Hit Probability Illustration 1

Lethal Radius = 3.75

Target Location Error = NORMAL $(0,5^2)$

Round-to-Round Error = NORMAL $(0,4^2)$

aim point (X = Y)	hit probability
0	0.430
1	0.427
2	0.404
3	0.369
4	0.325
5	0.268
6	0.205
7	0.160
8	0.108
9	0.070
10	0.047
11	0.028
12	0.015
13	0.011
14	0.004
15	0.002

TABLE II: Pattern Firing Hit Probability Illustration 2

Lethal Radius = 5

Target Location Error = NORMAL $(0,5^2)$

σ_2	aim at the center	pattern firing	
	probability	probability	best aim point
0.05	0.527	0.780	3.5
1.00	0.550	0.773	3.0
2.00	0.597	0.722	3.0
3.00	0.605	0.651	2.0
4.00	0.608	0.613	0.5
5.00	0.565	0.565	0.0
6.00	0.516	0.516	0.0

TABLE III: Comparison of Aiming at the Center and Pattern Firing

with $\sigma_1 = 5$ and $\sigma_2 = 1$ the best upper right aiming corner becomes (1.5, 1.5), (2.0, 2.0), (3.0, 3.0),(3.5, 3.5), (4.0, 4.0) as the lethal radius increases from 2.5 to 3.75, 5, 6.25, and 7.5.

As shown above, the exact relationship between all of the parameters of interest to maximize the expected hit probability is likely to be very complex. Therefore, a simplified general formula of hit probability that includes all of the parameters is not available.

In the next chapter we introduce another firing method called artillery registration which is often more efficient than pattern firing in the case where one observer provides feed back about miss distance between shots.

III. DISCUSSION OF ARTILLERY REGISTRATION METHOD

In the previous chapter, it was concluded that pattern firing is efficient when an observer is not available. This chapter will introduce a firing method called artillery registration (A.R.) that is used when noisy reports on miss distance are available.

In A.R., one aims his first shot at the estimated target position. After the first shot, the observer reports the distance between the impact point and the target location. But since the observer is inaccurate, he himself makes mistakes. In one dimension, suppose the marksman aims his first shot at 0 and the observer reports the target location is 10 to the right of the first hit. If the marksman believes the observer is accurate, he will choose the next aim point quite close to 10. If not, he will choose the next aim point quite close to the original aim point at 0. In any case, the next aim point will be somewhere on the line between 0 and 10. So the first problem is what point on the line between them should one locate his estimate, and secondarily, what is the variance of the target's position from this new estimate. That means what kind of random variable is the position of the target when the observer's report is given.

Let the X-coordinate of the target be a one-dimensional random variable X which is drawn from a normal distribution with mean μ and variance σ_T^2 . So the target location $X = \mu+E_1$

where E_1 represents the "target location" error governed by NORMAL $(0, \sigma_T^2)$. The marksman aims at μ and fires. The impact point Y is $\mu + E_2$ where E_2 represents "round-to-round" error governed by NORMAL $(0,\sigma_2^2)$. Then the reporter will observe the impact point Y and report the miss distance Z which is $X-Y+E_3 = X-(\mu+E_2) + E_3$ where E_3 represents the "report" error governed by NORMAL $(0,\sigma_3^2)$. Therefore, $E(Z|X) = X-E(Y|X) = X - \mu$ and the observation of miss distance Z is governed by the normal distribution with mean $X - \mu$ and variance $\sigma_Z^2 \equiv \sigma_2^2 + \sigma_3^2$ when X is given. So the question is, if X is initially normally distributed, what is the posterior distribution of X when Z is given?

The posterior density function $f_{X|Z}(x|z)$ can be determined from the joint density function $f_{X,Z}(x,z)$, since $f_{X|Z}(x|z) = K'' f_{X,Z}(x,z)$ in which k" represents some constant that does not depend on x. But

$$f_{X,Z}(x,z) = f_X(x) f_{Z|X}(z|x) = Ke^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma_T}\right)^2} e^{-\frac{1}{2}\left(\frac{Z-X+\mu}{\sigma_Z}\right)^2},$$

since $f_X(x)$ is NORMAL (μ,σ_T^2) and $f_{Z|X}(z|x)$ is NORMAL $(X-\mu,\sigma_Z^2)$. By equating powers of x^2 and x, we can write $f_{X,Z}(x,z)$ as

$$f_{X,Z}(x,z) = K' e^{-\frac{1}{2}\left(\frac{X-\mu'}{\sigma'}\right)^2}$$

Which is also a normal distribution with mean μ' and variance σ'^2. We have

$$\left(\frac{X-\mu}{\sigma_T}\right)^2 + \left(\frac{z-x+\mu}{\sigma_z}\right)^2 = \frac{X^2-2\mu x+\mu^2}{\sigma_T^2} + \frac{z^2-2zx+2z\mu+x^2-2\mu x+\mu^2}{\sigma_z^2}$$

$$= \left(\frac{1}{\sigma_T^2} + \frac{1}{\sigma_z^2}\right) x^2 - \left(\frac{\mu}{\sigma_T^2} + \frac{z+\mu}{\sigma_z^2}\right) 2x + \frac{2z\mu+\mu^2+z^2}{\sigma_z^2} + \frac{\mu^2}{\sigma_T^2}$$

We also have

$$\left(\frac{x-\mu'}{\sigma'}\right)^2 = \frac{1}{\sigma'^2}x^2 - \left(\frac{\mu'}{\sigma'^2}\right)2x + \frac{\mu'^2}{\sigma'^2}$$

Equating the coefficients of x^2 and x,

$$\frac{1}{\sigma'^2} = \frac{1}{\sigma_T^2} + \frac{1}{\sigma_z^2} \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{\mu'}{\sigma'^2} = \frac{\mu}{\sigma_T^2} + \frac{z+\mu}{\sigma_z^2} \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$\begin{aligned}
\mu' &= \frac{\mu}{\sigma_T^2}\sigma'^2 + \frac{z}{\sigma_z^2}\sigma'^2 + \frac{\mu}{\sigma_z^2}\sigma'^2 \\
&= \frac{\mu}{\sigma_T^2}\sigma'^2 + \frac{\mu}{\sigma_z^2}\sigma'^2 + \frac{z}{\sigma_z^2}\sigma'^2 \\
&= \mu\sigma'^2\left(\frac{1}{\sigma_T^2} + \frac{1}{\sigma_z^2}\right) + Z\frac{\sigma'^2}{\sigma_z^2} \\
&= \mu\frac{\sigma'^2}{\sigma'^2} + Z\frac{\sigma'^2}{\sigma_z^2} \\
&= \mu + Z\frac{\sigma'^2}{\sigma_z^2}
\end{aligned}$$

So the new mean μ' for X having observed Z depends upon Z. The new mean is the same as the old mean plus Z times some dimensionless constant. If Z is positive, the observer reported that X was greater than its mean. Consequently the new estimate of X should be greater than the old one. If Z is negative, the new estimate of X ought to be smaller than the old one.

Suppose for a moment that σ_z is infinite, then in the formula $\mu' = \mu + z\frac{\sigma'^2}{\sigma_z^2}$, the $z\frac{\sigma'^2}{\sigma_z^2}$ term becomes 0. The new estimate μ' is the same as the old estimate μ. That makes sense because $\sigma_z = \infty$ means that the reporter is useless. So the marksman should aim at the old estimate. On the other hand, if σ_z is very small, which means the observer is perfectly

accurate, it is not hard to show that $\mu' = Z + \mu$. This makes sense because when σ_z is 0, the observer's report is the exact value $X - \mu$. So the formula $\mu' = \mu + Z\frac{\sigma'^2}{\sigma_z^2}$ makes intuitive sense. Its use is what we refer to as "artillery registration."

Let

$$T = \frac{\sigma_z^2}{\sigma_T^2} = \frac{\sigma_2^2 + \sigma_3^2}{\sigma_T^2} \quad \text{for easy computation and}$$

$$T' = \frac{\sigma_z^2}{\sigma'^2} = \frac{\sigma_z^2}{\dfrac{\sigma_T^2\ \sigma_z^2}{\sigma_z^2+\sigma_T^2}} = \frac{\sigma_z^2\sigma_z^2 + \sigma_z^2\ \sigma_T^2}{\sigma_T^2\ \ \sigma_z^2} = \frac{\sigma_z^2}{\sigma_T^2} + 1 = T + 1$$

Then $\mu' = \mu + \dfrac{Z}{T'} = \mu + \dfrac{Z}{(T+1)}$

So far we have only discussed the case of one dimension. But the Y-coordinate has the same character as the X-coordinate, since errors are symmetric. Therefore, the above theory is exactly the same for the Y-coordinate.

With the above theory, the computer programming algorithm is shown in Figure 4. In the simulation program $\sigma_z^2 = \sigma_2^2 + \sigma_3^2 =$ "round-to-round" error plus "reporter" error, and the initial value of T is $\dfrac{\sigma_2^2 + \sigma_3^2}{\sigma_1^2}$. Let A_x represent the aiming point of the X-coordinate. The first shot is aimed at zero, since target location is assumed to be distributed NORMAL $(0,\sigma_1^2)$ initially. Then the impact point I_x of a first shot will be $A_x + E_2$, where E_2 represents the "round-to-round" error. If the target location is in the lethal radius of the impacted bomb, it is hit; if not, the observer will report the miss distance $R_x = X - I_x + E_3$, where E_3 is the "reporter" error which is drawn from NORMAL $(0,\sigma_3^2)$. Then the marksman will aim at the new aiming point A_x', that is old aiming point A_x plus $\dfrac{R_x}{(1 + T)}$.

As Figure 4 shows, the initial value of σ_T is σ_1 and the initial value of T is $\frac{\sigma_z^2}{\sigma_1^2}$. After every shot, the T value changes to T+1. So the procedure described above is essentially an application of Bayes' theorem over and over again. Each time through the loop, it aims at the most likely position of the target.

Table IV shows an example of computer results with lethal radius 5, target location error equals 5, and round-to-round error 2. As the table shows, the hit probability decreases when the reporter error increases.

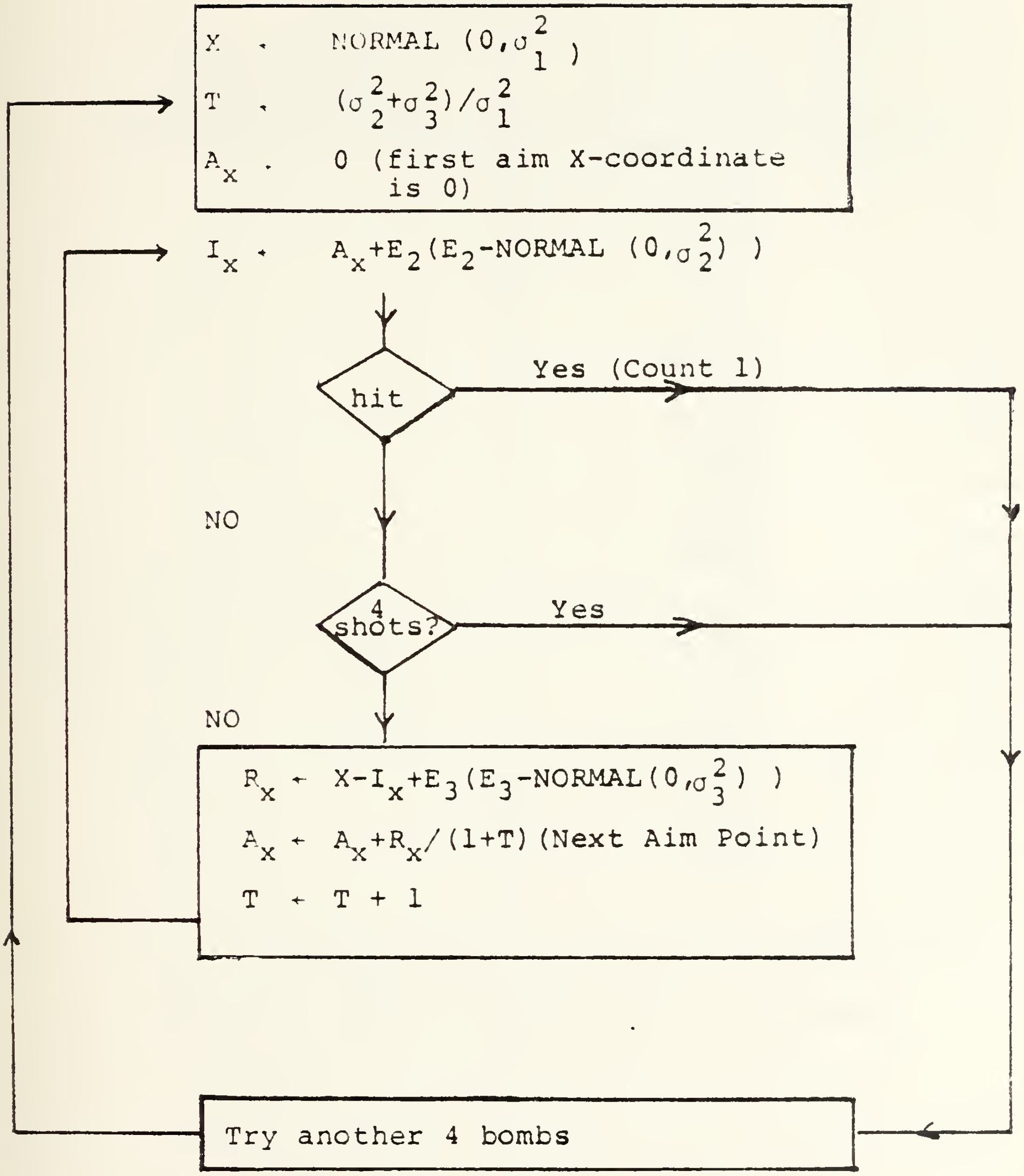

Figure 4: A.R. Firing Program Algorithm

Lethal Radius = 5

Target Location Error = NORMAL $(0, 5^2)$

Round-to-Round Error = NORMAL $(0, 2^2)$

(Report Error)	Hit Probability
0	0.999
1	0.998
2	0.992
3	0.980
4	0.952
5	0.916
6	0.883
7	0.841
8	0.808
9	0.786
10	0.755
11	0.742
12	0.723
13	0.709
14	0.695
15	0.689

TABLE IV: Hit Probability of A. R. Method

IV. COMPARISON OF PATTERN FIRING AND A.R. METHOD

In the previous two chapters, we discussed pattern firing and artillery registration. Figure 5, which is typical, illustrates the comparison of the two methods. The pattern firing probability is 0.773 and the A.R. probability is very high when σ_3 is small. As σ_3 increases, the A.R. hit probability decreases. At approximately $\sigma_3 = 8.8$, the hit probability of the two methods is the same. For larger values of σ_3, the A.R. method has a lower hit probability than the pattern firing method. With different values of R, σ_1, σ_2, and σ_3, the computer simulation shows that if the pattern firing method gives the best hit probability when aimed at the origin, then the A.R. method is always better. But in case the pattern is not null, then pattern firing is better when σ_3 is large.

In general neither method is optimal in all circumstances.

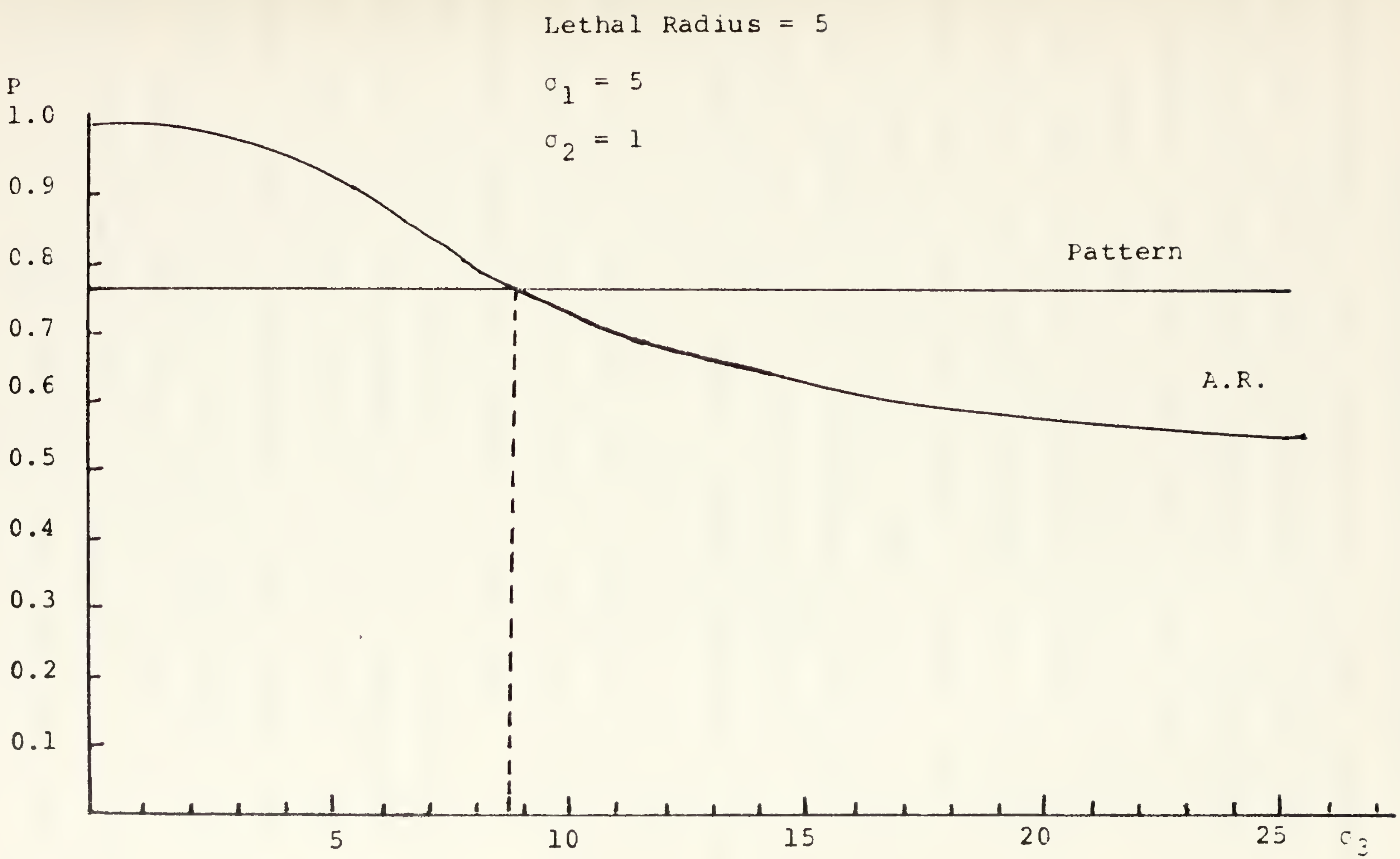

Figure 5: Graphical Comparison of Pattern and A.R.

V. EXPANDED ARTILLERY REGISTRATION METHOD

In this chapter, a new firing method called "expanded artillery registration" (E.A.R.) will be discussed. It is a feed back procedure plus some "artificial dispersion." The marksman follows the A.R. procedure, except that the round-to-round error is artificially increased to keep all the shots from bunching up. The reason for using artificial dispersion is shown in Figure 6.

In Figure 6, the triangle represents the real target location, the black circle is the best estimate of target location, the crosses represent the impact point when one fires without making artificial dispersion, and the squares are the impact points when one increases the round-to-round error artificially. The dotted circles represent lethal areas. Without artificial dispersion, the black circle area is over covered because the firing is too accurate. But the real target isn't covered at all. In the cases where one uses artificial dispersion, one bomb covers the target. So artificial dispersion is better in this case.

Table V shows the comparison of pattern firings, artillery registration (A.R.) and expanded artillery registration (E.A.R.). When the ratio of lethal radius to target location error ($\frac{R}{\sigma_1}$) is less than 0.5, the hit probability is low, pattern firing should aim at the origin, and A.R. is better than pattern firing or E.A.R. . Therefore, when the ratio $\frac{R}{\sigma_1}$ is less than 0.5, it is not worthwhile to make a table. So the table is made from the ratio $\frac{R}{\sigma_1} = 0.5$. $\frac{\sigma_3}{\sigma_1}$ in the 5th column represents the ratio

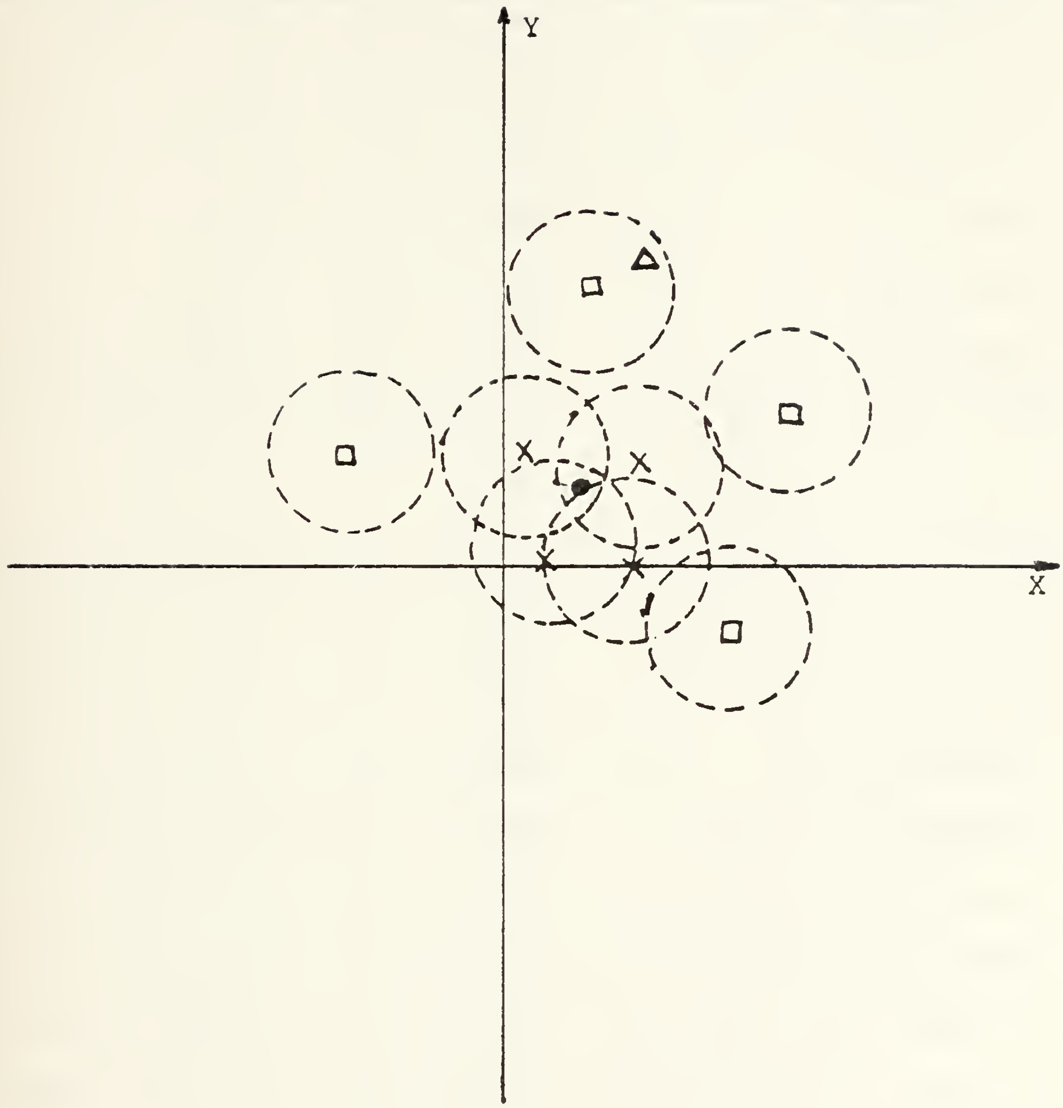

Figure 6. Illustration of Artificial Dispersion

R/σ_1	σ_2/σ_1	Pattern Aim Point (X/σ_1)	A.R. & Pattern Probability	σ_3/σ_1	E.A.R.	
					σ_2'/σ_1	Probability
0.5	0.01	0.4	0.345	2.0	0.4	0.377
0.5	0.2	0.4	0.312	2.8	0.4	0.330
0.5	0.4	0.4	0.280	6.0	0.6	0.286
0.5	0.6	0.0	0.254	∞	0.6	0.254
0.5	0.8	0.0	0.246	∞	0.8	0.246
0.5	1.0	0.0	0.206	∞	1.0	0.206
0.75	0.01	0.4	0.583	1.8	0.4	0.623
0.75	0.2	0.4	0.568	2.0	0.4	0.603
0.75	0.4	0.4	0.518	3.2	0.6	0.530
0.75	0.6	0.2	0.469	10.0	0.6	0.469
0.75	0.8	0.0	0.430	∞	0.8	0.430
0.75	1.0	0.0	0.394	∞	1.0	0.394
1.0	0.01	0.6	0.780	1.6	0.4	0.820
1.0	0.2	0.6	0.773	1.8	0.4	0.808
1.0	0.4	0.6	0.722	2.4	0.6	0.727
1.0	0.6	0.4	0.651	4.8	0.6	0.651
1.0	0.8	0.0	0.613	12.0	0.8	0.613
1.0	1.0	0.0	0.565	∞	1.0	0.565

TABLE V: Hit Probability Comparison of A.R. and E.A.R.

R/σ_1	σ_2/σ_1	Pattern Aim Point (X/σ_1)	A.R. & Pattern Probability	σ_3/σ_1	E.A.R.	
					σ_2'/σ_1	Probability
1.25	0.01	0.8	0.900	1.6	0.4	0.917
1.25	0.2	0.6	0.875	1.8	0.4	0.885
1.25	0.4	0.6	0.859	2.1	0.6	0.865
1.25	0.6	0.6	0.793	4.2	0.6	0.793
1.25	0.8	0.4	0.758	8.6	0.8	0.758
1.25	1.0	0.0	0.720	∞	1.0	0.720
1.5	0.01	0.8	0.950	1.6	0.4	0.956
1.5	0.2	0.8	0.948	1.7	0.4	0.952
1.5	0.4	0.8	0.938	1.9	0.4	0.938
1.5	0.6	0.6	0.887	3.2	0.6	0.887
1.5	0.8	0.2	0.849	7.8	0.8	0.849
1.5	1.0	0.0	0.820	∞	1.0	0.820

TABLE V: Hit Probability Comparison of A.R. and E.A.R.

of reporter error to target location error for which the A.R. hit probability is the same as that of pattern firing, with the common value being shown in the 4th column. As explained by Figure 5 in Chapter IV, the A.R. hit probability is higher than the pattern hit probability when the reporter error σ_3 is small. But the A.R. hit probability decreases as σ_3 increases. So when the ratio $\frac{\sigma_3}{\sigma_1}$ reaches the value in the 5th column in Table V, the two methods have the same hit probability. After that, pattern hit probability is higher than A.R. hit probability.

Therefore, the A.R. method is recommended rather than pattern firing or E.A.R. when $\frac{\sigma_3}{\sigma_1}$ is small. When $\frac{\sigma_3}{\sigma_1}$ is large, the pattern firing method is recommended rather than A.R. or E.A.R. On the other hand around $\frac{\sigma_3}{\sigma_1}$ in the 5th column, there is no great difference of hit probability between pattern firing and A.R. But expanded artillery registration gives higher probability than those of the other two methods. So $\frac{\sigma_3}{\sigma_1}$ where A.R. hit probability and pattern hit probability are the same is the best case for E.A.R.

The ratio $\frac{\sigma_2}{\sigma_1}$ represents the amount of artificial dispersion which gives the better probability in column 7. For example, when $\frac{R}{\sigma_1}$ is 0.5 and $\frac{\sigma_2}{\sigma_1}$ is 0.01, then pattern firing should aim at the 4 corners of the square, one of whose ratio of X-coordinate to σ_1 is 0.4. The hit probability is 0.345. On the other hand, the A.R. method gives the probability 0.345 when $\frac{\sigma_3}{\sigma_1}$ is 2.0. When $\frac{\sigma_3}{\sigma_1}$ is less than 2.0, the A.R. method gives a better hit probability. Given the condition that $\frac{\sigma_2}{\sigma_1}$ is 0.01 and $\frac{\sigma_3}{\sigma_1}$ is 2.0, one makes the artificial dispersion.

So one uses the round-to-round error ratio $(\frac{\sigma_2'}{\sigma_1})$ 0.4 instead of 0.01 and the probability is 0.377 which is higher than 0.345.

On the other hand when the ratio $\frac{\sigma_2}{\sigma_1}$ is 0.6, A.R. gives a hit probability that is always greater than but asymptotic to the pattern hit probability for large $\frac{\sigma_3}{\sigma_1}$, as evidenced by the ∞ symbol in the table. This means there is no necessity to make an artificial dispersion because round-to-round error is already big enough. Note that $\frac{\sigma_2'}{\sigma_1} = \frac{\sigma_2}{\sigma_1}$ in this case. Therefore, the computer simulation has indicated that artificial dispersion in artillery registration is sometimes a better method than either A.R. or pattern firing for scoring at least one hit with 4 bombs.

VI. MODIFIED ARTILLERY REGISTRATION METHOD

In the previous chapter, it was concluded that making some artificial dispersion in A.R. gives a better hit probability when the ratio $\frac{\sigma_2}{\sigma_1}$ is small. The E.A.R. uses round-to-round error which is larger than σ_2.

In this chapter, we will discuss another intentional dispersion method called modified artillery registration (M.A.R.). M.A.R. is artillery registration plus pattern firing. The round-to-round error direction is random in E.A.R. . But in the modified artillery registration, one decides the direction by choosing the corners of a square.

For example, to use 4 corners (1,1), (-1,1), (-1,-1), (1,-1) in M.A.R., the first shot aiming point will be the (1,1) which is the summation of (0,0) and(1,1). Then the observer will report the miss distance and the new estimate of the target location will be calculated. If the new estimate is (3,1), the second shot aiming point is (2,2), which is the sum of (3,1) and (-1,1). If the target is not hit and the new expected mean is (1,2), the third shot is aimed at the point (0,1), the sum of (1,2) and (-1,-1) and so on. So the modified artillery registration is the procedure which uses feed back plus pattern firing.

Figure 7 shows the comparison of four methods when all parameters except σ_3 are fixed. As the figure shows, modified artillery registration gives the best probability among the 4

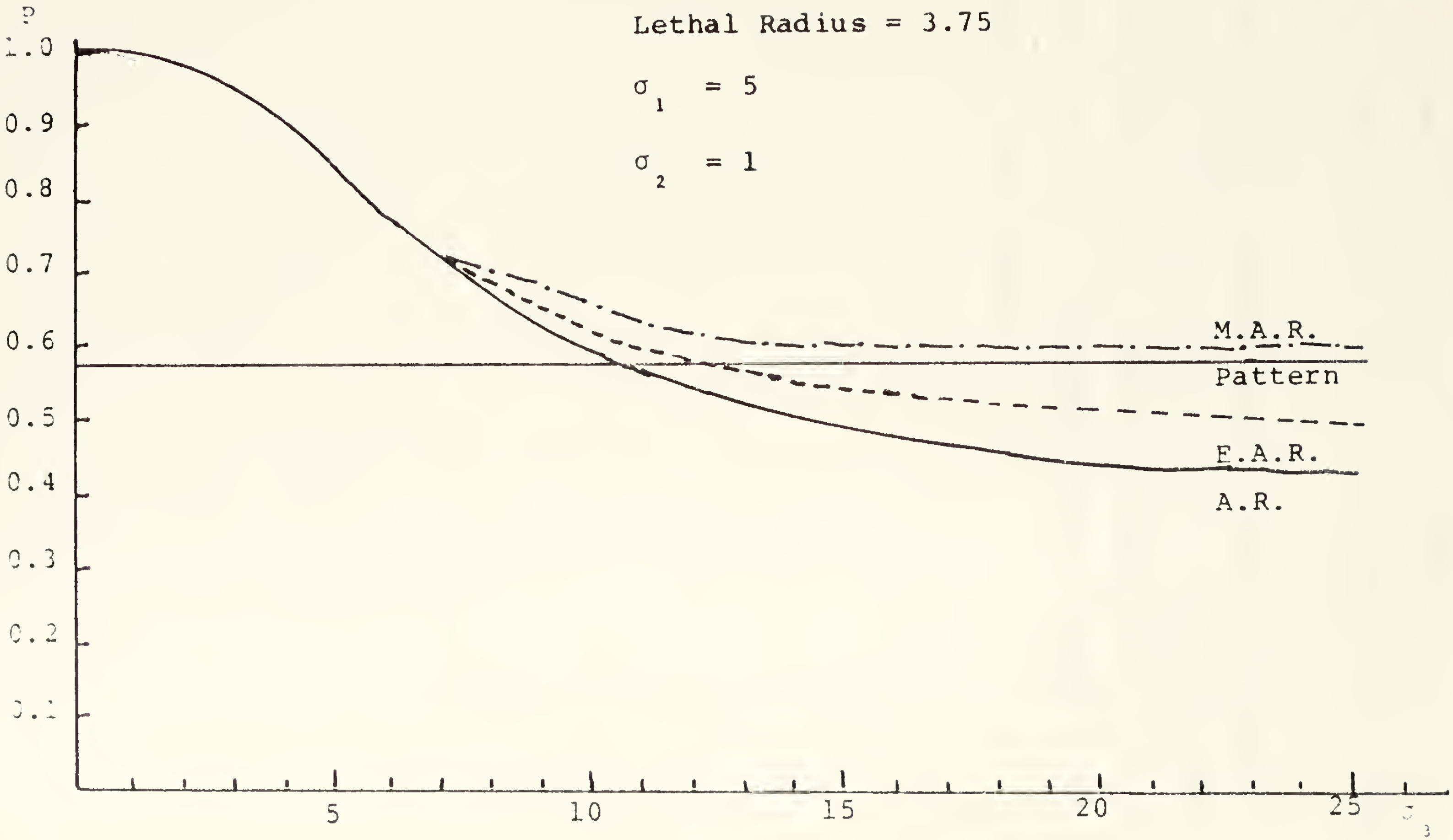

Figure 7: Graphical Comparison of Pattern, A.R., E.A.R., and M.A.R.

methods. Comparisons of hit probability of different methods with several parameters are presented in Table VI; M.A.R. always gives at least as high a hit probability as the other three methods.

As in Table V, $\frac{\sigma_3}{\sigma_1}$ represents the ratio of reporter error to aim point error where the pattern firing and artillery registration methods give the same hit probability. In the last column, the aim point represents the X-coordinate of one of the 4 corners of the square.

R/σ_1	σ_2/σ_1	σ_3/σ_1	Probability			M.A.R. Aim Point (X/σ_1)
			Pattern & A.R.	E.A.R.	M.A.R.	
0.5	0.01	2.0	0.345	0.377	0.400	0.4
0.5	0.2	2.8	0.312	0.330	0.350	0.4
0.5	0.4	6.0	0.280	0.286	0.311	0.4
0.5	0.6	∞	0.254	0.254	0.254	0.0
0.5	0.8	∞	0.246	0.246	0.246	0.0
0.5	1.0	∞	0.206	0.206	0.206	0.0
0.75	0.01	1.8	0.583	0.623	0.672	0.4
0.75	0.2	2.0	0.568	0.603	0.617	0.4
0.75	0.4	3.2	0.518	0.530	0.560	0.6
0.75	0.6	10.0	0.469	0.469	0.495	0.2
0.75	0.8	∞	0.430	0.430	0.430	0.0
0.75	1.0	∞	0.394	0.394	0.394	0.0
1.0	0.01	1.6	0.780	0.820	0.860	0.4
1.0	0.2	1.8	0.773	0.808	0.828	0.4
1.0	0.4	2.4	0.722	0.727	0.764	0.6
1.0	0.6	4.8	0.651	0.651	0.678	0.6
1.0	0.8	12.0	0.613	0.613	0.613	0.0
1.0	1.0	∞	0.565	0.565	0.565	0.0

TABLE VI: Hit Probability Comparison of A.R., E.A.R., and M.A.R.

R/σ_1	σ_2/σ_1	σ_3/σ_1	Probability			M.A.R. Aim Point (X/σ_1)
			Pattern & A.R.	E.A.R.	M.A.R.	
1.25	0.01	1.6	0.900	0.917	0.949	0.6
1.25	0.2	1.8	0.875	0.885	0.921	0.6
1.25	0.4	2.1	0.859	0.865	0.896	0.6
1.25	0.6	4.2	0.793	0.793	0.826	0.6
1.25	0.8	8.6	0.758	0.758	0.782	0.2
1.25	1.0	∞	0.720	0.720	0.720	0.0
1.5	0.01	1.6	0.950	0.956	0.983	0.6
1.5	0.2	1.7	0.948	0.952	0.974	0.8
1.5	0.4	1.9	0.938	0.938	0.962	0.6
1.5	0.6	3.2	0.887	0.887	0.916	0.6
1.5	0.8	7.8	01849	0.849	0.873	0.6
1.5	1.0	∞	0.820	0.820	0.820	0.0

TABLE VI: Hit Probability Comparison of A.R., E.A.R., and M.A.R.

VII. CONCLUSION AND RECOMMENDATION

The analysis made in the previous chapters indicated that neither the pattern firing method nor the artillery registration method is optimal and that intentional dispersion introduced to artillery registration by the square pattern will improve the chance of at least one hit.

Hit probability tables for pattern firing, artillery registration, and the modified artillery registration method calculated by computer simulation for $N = 4$ and various parameters combinations are given in the Appendix.

It is hoped that the analysis and models developed in this thesis will be of assistance to weapon system analysts, and that this work will generate an interest in additional investigations of the firing process.

APPENDIX: HIT PROBABILITY FOR N = 4

References for reading the table:

A. Sample Size = 10,000.

B. Confidence Interval of P in the table = $P \pm 0.0196\sqrt{P(1 - P)}$ with confidence coefficient 0.95.

For example, C.I. = 0.9 ± 0.0098 when P = 0.9

= 0.5 ± 0.0058 when P = 0.5.

C. List of Symbols

R = lethal radius or circular target radius

σ_1 = aiming error standard deviation

σ_2 = round-to-round error standard deviation

σ_3 = reporter error standard deviation

A.R. = Artillery Registration Method

M.A.R. = Modified Artillery Registration Method

D. Aim point represents the size of a square.

For example, if σ_1 = 2 and aim point value in the table is 0.4, the first aim point = σ_1 x aim point value = 0.8. Therefore, 4 bombs should be aimed at (0.8, 0.8), (0.8, -0.8), (-0.8, -0.8) and (-0.8, 0.8).

$R/\sigma_1 = 0.5$ $\sigma_2/\sigma_1 = 0.01$

PROBABILITY OF PATTERN = 0.345, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0
0.2	1.000	1.000	0
0.4	0.954	0.954	0
0.6	0.800	0.800	0
0.8	0.667	0.667	0
1.0	0.576	0.576	0
1.2	0.497	0.497	0
1.4	0.441	0.486	0.1-0.2
1.6	0.393	0.444	0.2-0.3
1.8	0.364	0.422	0.3-0.4
2.0	0.345	0.400	0.3-0.4
2.2	0.317	0.392	0.3-0.4
2.4	0.296	0.387	0.4-0.5
2.6	0.284	0.383	0.4-0.5
2.8	0.275	0.380	0.4-0.5
3.0	0.258	0.377	0.4-0.5
3.2	0.250	0.375	0.4-0.5
3.4	0.248	0.370	0.4-0.5
3.6	0.238	0.368	0.4-0.5
3.8	0.234	0.365	0.4-0.5
4.0	0.218	0.363	0.4-0.5
4.2	0.215	0.360	0.4-0.5
4.4	0.211	0.358	0.4-0.5
4.6	0.204	0.356	0.4-0.5
4.8	0.201	0.355	0.4-0.5
5.0	0.198	0.354	0.4-0.5

$R/\sigma_1 = 0.5$ $\sigma_2/\sigma_1 = 0.2$

PROBABILITY OF PATTERN = 0.312, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.998	0.998	0
0.2	0.990	0.990	0
0.4	0.917	0.917	0
0.6	0.789	0.789	0
0.8	0.667	0.667	0
1.0	0.577	0.577	0
1.2	0.507	0.507	0. -0.1
1.4	0.454	0.454	0. -0.1
1.6	0.409	0.429	0.1-0.2
1.8	0.392	0.413	0.1-0.2
2.0	0.369	0.404	0.2-0.3
2.2	0.349	0.385	0.2-0.3
2.4	0.333	0.373	0.3-0.4
2.6	0.321	0.358	0.3-0.4
2.8	0.309	0.350	0.4-0.5
3.0	0.295	0.345	0.4-0.5
3.2	0.285	0.342	0.4-0.5
3.4	0.279	0.338	0.4-0.5
3.6	0.269	0.336	0.4-0.5
3.8	0.266	0.335	0.4-0.5
4.0	0.262	0.334	0.4-0.5
4.2	0.259	0.333	0.4-0.5
4.4	0.255	0.333	0.4-0.5
4.6	0.248	0.332	0.4-0.5
4.8	0.245	0.332	0.4-0.5
5.0	0.241	0.332	0.4-0.5

$R/\sigma_1 = 0.5$ $\sigma_2/\sigma_1 = 0.4$

PROBABILITY OF PATTERN = 0.280, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.808	0.808	0
0.2	0.781	0.781	0
0.4	0.730	0.730	0
0.6	0.660	0.660	0
0.8	0.582	0.582	0
1.0	0.527	0.527	0
1.2	0.477	0.477	0
1.4	0.447	0.447	0
1.6	0.404	0.404	0. -0.1
1.8	0.392	0.392	0. -0.1
2.0	0.377	0.377	0. -0.1
2.2	0.364	0.372	0.1-0.2
2.4	0.352	0.363	0.1-0.2
2.6	0.339	0.354	0.1-0.2
2.8	0.330	0.350	0.2-0.3
3.0	0.324	0.345	0.2-0.3
3.2	0.321	0.339	0.2-0.3
3.4	0.313	0.335	0.2-0.3
3.6	0.306	0.327	0.2-0.3
3.8	0.304	0.325	0.2-0.3
4.0	0.302	0.323	0.2-0.3
4.2	0.301	0.320	0.3-0.4
4.4	0.299	0.319	0.3-0.4
4.6	0.297	0.317	0.3-0.4
4.8	0.295	0.315	0.3-0.4
5.0	0.293	0.312	0.3-0.4

$R/\sigma_1 = 0.5$ $\sigma_2/\sigma_1 = 0.6$

PROBABILITY OF PATTERN = 0.254, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.556	0.556	0
0.2	0.539	0.539	0
0.4	0.529	0.529	0
0.6	0.490	0.490	0
0.8	0.461	0.461	0
1.0	0.429	0.429	0
1.2	0.403	0.403	0
1.4	0.387	0.387	0
1.6	0.368	0.368	0
1.8	0.357	0.357	0
2.0	0.346	0.346	0
2.2	0.335	0.335	0
2.4	0.324	0.324	0
2.6	0.315	0.315	0
2.8	0.313	0.313	0
3.0	0.310	0.310	0
3.2	0.308	0.308	0
3.4	0.305	0.305	0
3.6	0.303	0.303	0
3.8	0.299	0.299	0
4.0	0.298	0.298	0
4.2	0.297	0.297	0
4.4	0.295	0.295	0
4.6	0.294	0.294	0
4.8	0.292	0.292	0
5.0	0.291	0.291	0

$R/\sigma_1 = 0.5$ $\sigma_2/\sigma_1 = 0.8$

PROBABILITY OF PATTERN = 0.246, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.395	0.395	0
0.2	0.393	0.393	0
0.4	0.379	0.379	0
0.6	0.364	0.364	0
0.8	0.345	0.345	0
1.0	0.332	0.332	0
1.2	0.321	0.321	0
1.4	0.316	0.316	0
1.6	0.312	0.312	0
1.8	0.307	0.307	0
2.0	0.294	0.294	0
2.2	0.290	0.290	0
2.4	0.286	0.286	0
2.6	0.283	0.283	0
2.8	0.281	0.281	0
3.0	0.278	0.278	0
3.2	0.274	0.274	0
3.4	0.272	0.272	0
3.6	0.270	0.270	0
3.8	0.268	0.268	0
4.0	0.267	0.267	0
4.2	0.266	0.266	0
4.4	0.265	0.265	0
4.6	0.264	0.264	0
4.8	0.262	0.262	0
5.0	0.261	0.261	0

$R/\sigma_1 = 0.5$ $\sigma_2/\sigma_1 = 1.0$

PROBABILITY OF PATTERN = 0.206, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.292	0.292	0
0.2	0.288	0.288	0
0.4	0.284	0.284	0
0.6	0.280	0.280	0
0.8	0.268	0.268	0
1.0	0.262	0.262	0
1.2	0.258	0.258	0
1.4	0.254	0.254	0
1.6	0.247	0.247	0
1.8	0.245	0.245	0
2.0	0.243	0.243	0
2.2	0.241	0.241	0
2.4	0.239	0.239	0
2.6	0.236	0.236	0
2.8	0.235	0.235	0
3.0	0.232	0.232	0
3.2	0.230	0.230	0
3.4	0.228	0.228	0
3.6	0.227	0.227	0
3.8	0.225	0.225	0
4.0	0.224	0.224	0
4.2	0.223	0.223	0
4.4	0.222	0.222	0
4.6	0.221	0.221	0
4.8	0.220	0.220	0
5.0	0.219	0.219	0

$R/\sigma_1 = 0.75$ $\sigma_2/\sigma_1 = 0.01$

PROBABILITY OF PATTERN = 0.583, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0
0.2	1.000	1.000	0
0.4	0.998	0.998	0
0.6	0.963	0.963	0
0.8	0.893	0.893	0
1.0	0.802	0.802	0. -0.1
1.2	0.733	0.743	0.2-0.3
1.4	0.671	0.710	0.3-0.4
1.6	0.623	0.688	0.3-0.4
1.8	0.587	0.672	0.4-0.5
2.0	0.549	0.647	0.5-0.6
2.2	0.518	0.642	0.5-0.6
2.4	0.495	0.638	0.5-0.6
2.6	0.481	0.634	0.5-0.6
2.8	0.463	0.632	0.5-0.6
3.0	0.449	0.631	0.5-0.6
3.2	0.436	0.631	0.5-0.6
3.4	0.422	0.630	0.5-0.6
3.6	0.414	0.630	0.5-0.6
3.8	0.401	0.629	0.5-0.6
4.0	0.389	0.629	0.5-0.6
4.2	0.386	0.628	0.6-0.7
4.4	0.381	0.628	0.6-0.7
4.6	0.372	0.627	0.6-0.7
4.8	0.370	0.627	0.6-0.7
5.0	0.367	0.627	0.6-0.7

$R/\sigma_1 = 0.75$ $\sigma_2/\sigma_1 = 0.2$

PROBABILITY OF PATTERN = 0.568, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0
0.2	0.999	0.999	0
0.4	0.993	0.993	0
0.6	0.953	0.953	0
0.8	0.881	0.881	0
1.0	0.815	0.815	0
1.2	0.746	0.746	0. -0.1
1.4	0.693	0.693	0. -0.1
1.6	0.648	0.663	0.1-0.2
1.8	0.602	0.643	0.2-0.3
2.0	0.568	0.617	0.3-0.4
2.2	0.546	0.612	0.4-0.5
2.4	0.524	0.605	0.5-0.6
2.6	0.513	0.602	0.5-0.6
2.8	0.491	0.600	0.5-0.6
3.0	0.483	0.596	0.5-0.6
3.2	0.465	0.592	0.5-0.6
3.4	0.455	0.589	0.5-0.6
3.6	0.449	0.588	0.5-0.6
3.8	0.441	0.586	0.5-0.6
4.0	0.427	0.584	0.5-0.6
4.2	0.425	0.584	0.5-0.6
4.4	0.422	0.583	0.5-0.6
4.6	0.412	0.582	0.5-0.6
4.8	0.409	0.581	0.5-0.6
5.0	0.404	0.580	0.5-0.6

$R/\sigma_1 = 0.75$ $\sigma_2/\sigma_1 = 0.4$

PROBABILITY OF PATTERN = 0.518, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.978	0.978	0
0.2	0.970	0.970	0
0.4	0.940	0.940	0
0.6	0.903	0.903	0
0.8	0.845	0.845	0
1.0	0.783	0.783	0
1.2	0.738	0.738	0. -0.1
1.4	0.696	0.696	0. -0.1
1.6	0.659	0.659	0. -0.1
1.8	0.623	0.650	0.2-0.3
2.0	0.603	0.627	0.2-0.3
2.2	0.578	0.611	0.2-0.3
2.4	0.566	0.599	0.3-0.4
2.6	0.547	0.580	0.3-0.4
2.8	0.533	0.569	0.3-0.4
3.0	0.528	0.562	0.4-0.5
3.2	0.512	0.560	0.4-0.5
3.4	0.510	0.558	0.4-0.5
3.6	0.500	0.556	0.4-0.5
3.8	0.498	0.553	0.4-0.5
4.0	0.498	0.550	0.4-0.5
4.2	0.493	0.546	0.5-0.6
4.4	0.485	0.544	0.5-0.6
4.6	0.480	0.542	0.5-0.6
4.8	0.475	0.540	0.5-0.6
5.0	0.471	0.540	0.5-0.6

$R/\sigma_1 = 0.75$ $\sigma_2/\sigma_1 = 0.6$

PROBABILITY OF PATTERN = 0.469, AIM POINT = 0.2			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R.AIM POINT
0	0.836	0.836	0
0.2	0.830	0.830	0
0.4	0.811	0.811	0
0.6	0.774	0.774	0
0.8	0.731	0.731	0
1.0	0.715	0.715	0
1.2	0.675	0.675	0
1.4	0.646	0.646	0
1.6	0.623	0.623	0
1.8	0.604	0.604	0
2.0	0.591	0.591	0. -0.1
2.2	0.570	0.570	0. -0.1
2.4	0.558	0.558	0. -0.1
2.6	0.545	0.545	0. -0.1
2.8	0.538	0.538	0. -0.1
3.0	0.533	0.533	0. -0.1
3.2	0.529	0.530	0.1-0.2
3.4	0.524	0.528	0.1-0.2
3.6	0.516	0.524	0.1-0.2
3.8	0.511	0.522	0.1-0.2
4.0	0.502	0.520	0.1-0.2
4.2	0.500	0.518	0.1-0.2
4.4	0.499	0.515	0.2-0.3
4.6	0.498	0.513	0.2-0.3
4.8	0.497	0.512	0.2-0.3
5.0	0.496	0.511	0.2-0.3

$R/\sigma_1 = 0.75$ $\sigma_2/\sigma_1 = 0.8$

PROBABILITY OF PATTERN = 0.430, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.666	0.666	0
0.2	0.661	0.661	0
0.4	0.656	0.656	0
0.6	0.642	0.642	0
0.8	0.622	0.622	0
1.0	0.589	0.589	0
1.2	0.574	0.574	0
1.4	0.563	0.563	0
1.6	0.559	0.559	0
1.8	0.549	0.549	0
2.0	0.526	0.526	0.
2.2	0.520	0.520	0
2.4	0.515	0.515	0
2.6	0.510	0.510	0
2.8	0.498	0.498	0
3.0	0.495	0.495	0
3.2	0.493	0.493	0
3.4	0.491	0.491	0
3.6	0.483	0.483	0
3.8	0.480	0.480	0
4.0	0.479	0.479	0
4.2	0.478	0.478	0
4.4	0.475	0.475	0
4.6	0.472	0.472	0
4.8	0.468	0.468	0
5.0	0.467	0.467	0

$R/\sigma_1 = 0.75$ $\sigma_2/\sigma_1 = 1.0$

PROBABILITY OF PATTERN = 0.394, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.532	0.532	0
0.2	0.528	0.528	0
0.4	0.524	0.524	0
0.6	0.519	0.519	0
0.8	0.510	0.510	0
1.0	0.496	0.496	0
1.2	0.482	0.482	0
1.4	0.471	0.471	0
1.6	0.462	0.462	0
1.8	0.459	0.459	0
2.0	0.457	0.457	0
2.2	0.453	0.453	0
2.4	0.448	0.448	0
2.6	0.442	0.442	0
2.8	0.437	0.437	0
3.0	0.434	0.434	0
3.2	0.432	0.432	0
3.4	0.429	0.429	0
3.6	0.427	0.427	0
3.8	0.425	0.425	0
4.0	0.422	0.422	0
4.2	0.420	0.420	0
4.4	0.419	0.419	0
4.6	0.419	0.419	0
4.8	0.418	0.418	0
5.0	0.418	0.418	0

$R/\sigma_1 = 1.0$ $\sigma_2/\sigma_1 = 0.01$

PROBABILITY OF PATTERN = 0.780, AIM POINT = 0.6			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0
0.2	1.000	1.000	0
0.4	1.000	1.000	0
0.6	0.994	0.994	0
0.8	0.974	0.974	0. -0.1
1.0	0.925	0.925	0. -0.1
1.2	0.876	0.901	0.2-0.3
1.4	0.827	0.871	0.3-0.4
1.6	0.780	0.860	0.4-0.5
1.8	0.747	0.848	0.5-0.6
2.0	0.712	0.836	0.5-0.6
2.2	0.683	0.829	0.5-0.6
2.4	0.657	0.821	0.5-0.6
2.6	0.636	0.815	0.6-0.7
2.8	0.622	0.813	0.6-0.7
3.0	0.604	0.812	0.6-0.7
3.2	0.586	0.810	0.6-0.7
3.4	0.578	0.808	0.6-0.7
3.6	0.566	0.805	0.7-0.8
3.8	0.552	0.803	0.7-0.8
4.0	0.548	0.800	0.7-0.8
4.2	0.540	0.796	0.7-0.8
4.4	0.530	0.794	0.7-0.8
4.6	0.526	0.794	0.7-0.8
4.8	0.521	0.793	0.7-0.8
5.0	0.513	0.793	0.7-0.8

$R/\sigma_1 = 1.0$ $\sigma_2/\sigma_1 = 0.2$

PROBABILITY OF PATTERN = 0.773, AIM POINT = 0.6			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0
0.2	1.000	1.000	0
0.4	0.999	0.999	0
0.6	0.993	0.993	0
0.8	0.968	0.968	0 -0.1
1.0	0.927	0.936	0.2-0.3
1.2	0.882	0.897	0.3-0.4
1.4	0.843	0.867	0.3-0.4
1.6	0.795	0.833	0.4-0.5
1.8	0.764	0.828	0.4-0.5
2.0	0.730	0.823	0.5-0.6
2.2	0.705	0.815	0.5-0.6
2.4	0.684	0.799	0.5-0.6
2.6	0.667	0.795	0.6-0.7
2.8	0.649	0.791	0.6-0.7
3.0	0.638	0.789	0.6-0.7
3.2	0.627	0.787	0.6-0.7
3.4	0.641	0.785	0.6-0.7
3.6	0.603	0.784	0.7-0.8
3.8	0.591	0.783	0.7-0.8
4.0	0.584	0.783	0.7-0.8
4.2	0.579	0.782	0.7-0.8
4.4	0.571	0.782	0.7-0.8
4.6	0.564	0.781	0.7-0.8
4.8	0.559	0.781	0.7-0.8
5.0	0.557	0.781	0.7-0.8

$R/\sigma_1 = 1.0$ $\sigma_2/\sigma_1 = 0.4$

PROBABILITY OF PATTERN = 0.722, AIM POINT = 0.6			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R.AIM POINT
0	0.999	0.999	0
0.2	0.998	0.998	0
0.4	0.992	0.992	0
0.6	0.980	0.980	0
0.8	0.952	0.952	0
1.0	0.916	0.916	0
1.2	0.883	0.883	0. -0.1
1.4	0.841	0.846	0.1-0.2
1.6	0.820	0.827	0.3-0.4
1.8	0.786	0.802	0.3-0.4
2.0	0.755	0.779	0.3-0.4
2.2	0.742	0.771	0.4-0.5
2.4	0.723	0.764	0.4-0.5
2.6	0.709	0.752	0.5-0.6
2.8	0.695	0.743	0.5-0.6
3.0	0.689	0.737	0.5-0.6
3.2	0.670	0.734	0.5-0.6
3.4	0.661	0.730	0.5-0.6
3.6	0.656	0.725	0.6-0.7
3.8	0.650	0.725	0.6-0.7
4.0	0.643	0.724	0.6-0.7
4.2	0.638	0.723	0.6-0.7
4.4	0.634	0.723	0.6-0.7
4.6	0.629	0.722	0.6-0.7
4.8	0.625	0.722	0.6-0.7
5.0	0.622	0.722	0.6-0.7

$R/\sigma_1 = 1.0$ $\sigma_2/\sigma_1 = 0.6$

PROBABILITY OF PATTERN = 0.651, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.962	0.962	0
0.2	0.957	0.957	0
0.4	0.948	0.948	0
0.6	0.931	0.931	0
0.8	0.899	0.899	0
1.0	0.875	0.875	0
1.2	0.848	0.848	0
1.4	0.828	0.828	0
1.6	0.806	0.806	0
1.8	0.777	0.777	0
2.0	0.761	0.761	0
2.2	0.745	0.745	0. -0.1
2.4	0.737	0.737	0. -0.1
2.6	0.725	0.725	0. -0.1
2.8	0.704	0.711	0.1-0.2
3.0	0.699	0.707	0.2-0.3
3.2	0.692	0.704	0.2-0.3
3.4	0.687	0.701	0.2-0.3
3.6	0.679	0.698	0.3-0.4
3.8	0.674	0.694	0.3-0.4
4.0	0.669	0.691	0.4-0.5
4.2	0.664	0.688	0.4-0.5
4.4	0.661	0.684	0.4-0.5
4.6	0.657	0.681	0.5-0.6
4.8	0.654	0.678	0.5-0.6
5.0	0.649	0.674	0.5-0.6

$R/\sigma_1 = 1.0$ $\sigma_2/\sigma_1 = 0.8$

PROBABILITY OF PATTERN = 0.613, AIM POINT =0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R.PROBABILITY	M.A.R. AIM POINT
0	0.864	0.864	0
0.2	0.855	0.855	0
0.4	0.848	0.848	0
0.6	0.833	0.833	0
0.8	0.820	0.820	0
1.0	0.805	0.805	0
1.2	0.781	0.781	0
1.4	0.761	0.761	0
1.6	0.749	0.749	0
1.8	0.729	0.729	0
2.0	0.722	0.722	0
2.2	0.711	0.711	0
2.4	0.700	0.700	0
2.6	0.692	0.692	0
2.8	0.683	0.683	0
3.0	0.678	0.678	0
3.2	0.674	0.674	0
3.4	0.668	0.668	0
3.6	0.666	0.666	0
3.8	0.661	0.661	0
4.0	0.657	0.657	0
4.2	0.655	0.655	0
4.4	0.652	0.652	0
4.6	0.647	0.647	0
4.8	0.645	0.645	0
5.0	0.644	0.644	0

$R/\sigma_1 = 1.0$ $\sigma_2/\sigma_1 = 1.0$

PROBABILITY OF PATTERN = 0.565, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.743	0.743	0
0.2	0.740	0.740	0
0.4	0.737	0.737	0
0.6	0.728	0.728	0
0.8	0.720	0.720	0
1.0	0.705	0.705	0
1.2	0.690	0.690	0
1.4	0.678	0.678	0
1.6	0.673	0.673	0
1.8	0.670	0.670	0
2.0	0.664	0.664	0
2.2	0.652	0.652	0
2.4	0.645	0.645	0
2.6	0.640	0.640	0
2.8	0.637	0.637	0
3.0	0.630	0.630	0
3.2	0.625	0.625	0
3.4	0.620	0.620	0
3.6	0.618	0.618	0
3.8	0.614	0.614	0
4.0	0.612	0.612	0
4.2	0.610	0.610	0
4.4	0.608	0.608	0
4.6	0.606	0.606	0
4.8	0.604	0.604	0
5.0	0.603	0.603	0

$R/\sigma_1 = 1.25$ $\sigma_2/\sigma_1 = 0.01$

PROBABILITY OF PATTERN = 0.900, AIM POINT = 0.8			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0. -0.2
0.2	1.000	1.000	0. -0.2
0.4	1.000	1.000	0. -0.2
0.6	0.999	0.999	0. -0.2
0.8	0.995	0.995	0. -0.2
1.0	0.978	0.978	0. -0.2
1.2	0.955	0.961	0.2-0.3
1.4	0.920	0.955	0.4-0.5
1.6	0.895	0.949	0.5-0.6
1.8	0.861	0.935	0.6-0.7
2.0	0.840	0.928	0.6-0.7
2.2	0.813	0.920	0.6-0.7
2.4	0.797	0.918	0.7-0.8
2.6	0.774	0.915	0.7-0.8
2.8	0.762	0.914	0.7-0.8
3.0	0.743	0.912	0.7-0.8
3.2	0.733	0.910	0.7-0.8
3.4	0.721	0.907	0.7-0.8
3.6	0.712	0.906	0.7-0.8
3.8	0.708	0.905	0.7-0.8
4.0	0.697	0.904	0.8-0.9
4.2	0.689	0.904	0.8-0.9
4.4	0.684	0.903	018-0.9
4.6	0.676	0.903	0.8-0.9
4.8	0.672	0.902	0.8-0.9
5.0	0.665	0.902	0.8-0.9

$R/\sigma_1 = 1.25$ $\sigma_2/\sigma_1 = 0.2$

PROBABILITY OF PATTERN = 0.875, AIM POINT = 0.6			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0. -0.2
0.2	1.000	1.000	0. -0.2
0.4	1.000	1.000	0. -0.2
0.6	0.999	0.999	0. -0.2
0.8	0.993	0.993	0. -0.2
1.0	0.978	0.978	0. -0.2
1.2	0.956	0.960	0.2-0.3
1.4	0.925	0.945	0.3-0.4
1.6	0.899	0.932	0.4-0.5
1.8	0.878	0.921	0.5-0.6
2.0	0.851	0.918	0.6-0.7
2.2	0.828	0.914	0.6-0.7
2.4	0.806	0.912	0.7-0.8
2.6	0.791	0.908	0.7-0.8
2.8	0.778	0.906	0.7-0.8
3.0	0.768	0.904	0.7-0.8
3.2	0.759	0.903	0.7-0.8
3.4	9.751	0.903	0.7-0.8
3.6	0.741	0.902	0.7-0.8
3.8	0.732	0.902	0.7-0.8
4.0	0.726	0.901	0.7-0.8
4.2	0.718	0.901	0.8-0.9
4.4	0.714	0.900	0.8-0.9
4.6	0.711	0.900	0.8-0.9
4.8	0.707	0.899	0.8-0.9
5.0	0.705	0.899	0.8-0.9

$R/\sigma_1 = 1.25$ $\sigma_2/\sigma_1 = 0.4$

PROBABILITY OF PATTERN = 0.859, AIM POINT = 0.6			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0. -0.2
0.2	1.000	1.000	0. -0.2
0.4	0.999	0.999	0. -0.2
0.6	0.998	0.998	0. -0.2
0.8	0.990	0.990	0. -0.2
1.0	0.970	0.970	0. -0.2
1.2	0.953	0.953	0. -0.2
1.4	0.927	0.932	0.2-0.3
1.6	0.907	0.925	0.3-0.4
1.8	0.885	0.915	0.3-0.4
2.0	0.866	0.911	0.4-0.5
2.2	0.851	0.891	0.5-0.6
2.4	0.834	0.886	0.5-0.6
2.6	0.822	0.882	0.6-0.7
2.8	0.813	0.880	0.6-0.7
3.0	0.804	0.877	0.6-0.7
3.2	0.795	0.876	0.7-0.8
3.4	0.786	0.874	0.7-0.8
3.6	0.779	0.872	0.7-0.8
3.8	0.773	0.871	0.7-0.8
4.0	0.768	0.869	0.7-0.8
4.2	0.764	0.867	0.7-0.8
4.4	0.760	0.865	0.7-0.8
4.6	0.758	0.863	0.7-0.8
4.8	0.652	0.862	0.7-0.8
5.0	0.745	0.861	0.7-0.8

$R/\sigma_1 = 1.25$ $\sigma_2/\sigma_1 = 0.6$

PROBABILITY OF PATTERN = 0.793, AIM POINT = 0.6			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.993	0.993	0
0.2	0.992	0.992	0
0.4	0.987	0.987	0
0.6	0.982	0.982	0
0.8	0.970	0.970	0
1.0	0.954	0.954	0
1.2	0.938	0.938	0
1.4	0.921	0.921	0
1.6	0.904	0.912	0. -0.1
1.8	0.888	0.901	0.1-0.2
2.0	0.873	0.886	0.2-0.3
2.2	0.859	0.876	0.2-0.3
2.4	0.848	0.863	0.2-0.3
2.6	0.836	0.856	0.3-0.4
2.8	0.827	0.848	0.3-0.4
3.0	0.818	0.842	0.3-0.4
3.2	0.812	0.836	0.3-0.4
3.4	0.806	0.834	0.4-0.5
3.6	0.802	0.832	0.4-0.5
3.8	0.797	0.830	0.4-0.5
4.0	0.795	0.828	0.5-0.6
4.2	0.793	0.826	0.5-0.6
4.4	0.791	0.825	0.5-0.6
4.6	0.785	0.823	0.6-0.7
4.8	0.782	0.821	0.6-0.7
5.0	0.779	0.820	0.6-0.7

$R/\sigma_1 = 1.25$ $\sigma_2/\sigma_1 = 0.8$

PROBABILITY OF PATTERN = 0.758, AIM POINT = 0.4			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.955	0.955	0
0.2	0.952	0.952	0
0.4	0.950	0.950	0
0.6	0.936	0.936	0
0.8	0.928	0.928	0
1.0	0.916	0.916	0
1.2	0.906	0.906	0
1.4	0.889	0.889	0
1.6	0.881	0.881	0
1.8	0.864	0.864	0
2.0	0.851	0.851	0. -0.1
2.2	0.839	0.839	0. -0.1
2.4	0.827	0.836	0.1-0.2
2.6	0.821	0.832	0.1-0.2
2.8	0.815	0.830	0.1-0.2
3.0	0.807	0.825	0.1-0.2
3.2	0.804	0.822	0.1-0.2
3.4	0.801	0.820	0.1-0.2
3.6	0.796	0.818	0.1-0.2
3.8	0.792	0.816	0.1-0.2
4.0	0.790	0.813	0.1-0.2
4.2	0.788	0.811	0.1-0.2
4.4	0.786	0.810	0.1-0.2
4.6	0.782	0.808	0.1-0.2
4.8	0.778	0.807	0.1-0.2
5.0	0.776	0.805	0.1-0.2

$R/\sigma_1 = 1.25$ $\sigma_2/\sigma_1 = 1.0$

PROBABILITY OF PATTERN = 0.720, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.878	0.878	0
0.2	0.874	0.874	0
0.4	0.870	0.870	0
0.6	0.866	0.866	0
0.8	0.857	0.857	0
1.0	0.848	0.848	0
1.2	0.839	0.839	0
1.4	0.827	0.827	0
1.6	0.817	0.817	0
1.8	0.807	0.807	0
2.0	0.801	0.801	0
2.2	0.796	0.796	0
2.4	0.790	0.790	0
2.6	0.786	0.786	0
2.8	0.782	0.782	0
3.0	0.777	0.777	0
3.2	0.768	0.768	0
3.4	0.765	0.765	0
3.6	0.762	0.762	0
3.8	0.760	0.760	0
4.0	0.759	0.759	0
4.2	0.758	0.758	0
4.4	0.757	0.757	0
4.6	0.756	0.756	0
4.8	0.755	0.755	0
5.0	0.754	0.754	0

$R/\sigma_1 = 1.5$ $\sigma_2/\sigma_1 = 0.01$

PROBABILITY OF PATTERN = 0.950, AIM POINT = 0.8			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0. -0.4
0.2	1.000	1.000	0. -0.4
0.4	1.000	1.000	0. -0.4
0.6	1.000	1.000	0. -0.4
0.8	0.999	0.999	0. -0.4
1.0	0.996	0.996	0. -0.4
1.2	0.985	0.985	0. -0.4
1.4	0.971	0.984	0.5-0.6
1.6	0.951	0.983	0.6-0.7
1.8	0.938	0.981	0.7-0.8
2.0	0.918	0.978	0.7-0.8
2.2	0.899	0.976	0.8-0.9
2.4	0.886	0.973	0.8-0.9
2.6	0.871	0.972	0.8-0.9
2.8	0.862	0.970	0.8-0.9
3.0	0.846	0.969	0.8-0.9
3.2	0.836	0.969	0.8-0.9
3.4	0.829	0.968	0.9-1.0
3.6	0.821	0.968	0.9-1.0
3.8	0.813	0.967	0.9-1.0
4.0	0.809	0.967	0.9-1.0
4.2	0.803	0.967	0.9-1.0
4.4	0.797	0.966	0.9-1.0
4.6	0.790	0.966	0.9-1.0
4.8	0.785	0.966	0.9-1.0
5.0	0.778	0.965	0.9-1.0

$R/\sigma_1 = 1.5$ $\sigma_2/\sigma_1 = 0.2$

PROBABILITY OF PATTERN = 0.948, AIM POINT = 0.8			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0. -0.2
0.2	1.000	1.000	0. -0.2
0.4	1.000	1.000	0. -0.2
0.6	1.000	1.000	0. -0.2
0.8	0.999	0.999	0. -0.2
1.0	0.993	0.997	0.2-0.3
1.2	0.985	0.989	0.2-0.3
1.4	0.972	0.982	0.3-0.4
1.6	0.954	0.976	0.4-0.5
1.8	0.935	0.972	0.6-0.7
2.0	0.921	0.970	0.7-0.8
2.2	0.913	0.968	0.7-0.8
2.4	0.899	0.966	0.8-0.9
2.6	0.882	0.965	0.8-0.9
2.8	0.873	0.964	0.8-0.9
3.0	0.865	0.964	0.8-0.9
3.2	0.859	0.963	0.8-0.9
3.4	0.849	0.963	0.9-1.0
3.6	0.837	0.962	0.9-1.0
3.8	0.832	0.962	0.9-1.0
4.0	0.826	0.961	0.9-1.0
4.2	0.821	0.961	0.9-1.0
4.4	0.817	0.961	0.9-1.0
4.6	0.815	0.960	0.9-1.0
4.8	0.813	0.960	0.9-1.0
5.0	0.811	0.960	0.9-1.0

R/σ_1 = 1.5 σ_2/σ_1 = 0.4

PROBABILITY OF PATTERN = 0.938, AIM POINT = 0.8			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0. -0.2
0.2	1.000	1.000	0. -0.2
0.4	1.000	1.000	0. -0.2
0.6	0.999	0.999	0. -0.2
0.8	0.998	0.998	0. -0.2
1.0	0.993	0.993	0. -0.2
1.2	0.984	0.984	0. -0.2
1.4	0.971	0.980	0.2-0.3
1.6	0.956	0.974	0.4-0.5
1.8	0.949	0.967	0.5-0.6
2.0	0.932	0.960	0.5-0.6
2.2	0.920	0.958	0.6-0.7
2.4	0.907	0.955	0.6-0.7
2.6	0.895	0.952	0.6-0.7
2.8	0.890	0.949	0.6-0.7
3.0	0.884	0.947	0.7-0.8
3.2	0.877	0.945	0.7-0.8
3.4	0.872	0.943	0.8-0.9
3.6	0.866	0.942	0.8-0.9
3.8	0.860	0.941	0.8-0.9
4.0	0.855	0.940	0.8-0.9
4.2	0.850	0.938	0.9-1.0
4.4	0.847	0.938	0.9-1.0
4.6	0.845	0.936	0.9-1.0
4.8	0.844	0.935	0.9-1.0
5.0	0.842	0.934	0.9-1.0

$R/\sigma_1 = 1.5$ $\sigma_2/\sigma_1 = 0.6$

PROBABILITY OF PATTERN = 0.887, AIM POINT = 0.6			
σ_3/σ_1	A.R.PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	1.000	1.000	0
0.2	1.000	1.000	0
0.4	0.998	0.998	0
0.6	0.997	0.997	0
0.8	0.992	0.992	0
1.0	0.987	0.987	0
1.2	0.976	0.976	0
1.4	0.969	0.969	0. -0.2
1.6	0.954	0.962	0.2-0.3
1.8	0.942	0.958	0.2-0.3
2.0	0.930	0.952	0.3-0.4
2.2	0.921	0.944	0.4-0.5
2.4	0.911	0.937	0.4-0.5
2.6	0.909	0.931	0.5-0.6
2.8	0.900	0.925	0.5-0.6
3.0	0.889	0.920	0.5-0.6
3.2	0.883	0.916	0.6-0.7
3.4	0.878	0.914	0.6-0.7
3.6	0.875	0.913	0.6-0.7
3.8	0.871	0.911	0.6-0.7
4.0	0.868	0.910	0.6-0.7
4.2	0.864	0.908	0.6-0.7
4.4	0.862	0.907	0.7-0.8
4.6	0.860	0.906	0.7-0.8
4.8	0.858	0.905	0.7-0.8
5.0	0.856	0.904	0.7-0.8

$R/\sigma_1 = 1.5$ $\sigma_2/\sigma_1 = 0.8$

PROBABILITY OF PATTERN = 0.849, AIM POINT = 0.2			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.989	0.989	0
0.2	0.988	0.988	0
0.4	0.987	0.987	0
0.6	0.982	0.982	0
0.8	0.976	0.976	0
1.0	0.969	0.969	0
1.2	0.961	0.961	0
1.4	0.953	0.953	0
1.6	0.942	0.942	0. -0.1
1.8	0.931	0.931	0. -0.1
2.0	0.922	0.928	0.1-0.2
2.2	0.915	0.925	0.1-0.2
2.4	0.910	0.917	0.1-0.2
2.6	0.904	0.913	0.1-0.2
2.8	0.898	0.911	0.2-0.3
3.0	0.892	0.906	0.2-0.3
3.2	0.885	0.902	0.3-0.4
3.4	0.880	0.900	0.3-0.4
3.6	0.878	0.897	0.3-0.4
3.8	0.877	0.895	0.3-0.4
4.0	0.875	0.889	0.4-0.5
4.2	0.873	0.886	0.4-0.5
4.4	0.870	0.884	0.5-0.6
4.6	0.868	0.883	0.5-0.6
4.8	0.866	0.882	0.5-0.6
5.0	0.864	0.881	0.5-0.6

$R/\sigma_1 = 1.5$ $\sigma_2/\sigma_1 = 1.0$

PROBABILITY OF PATTERN = 0.820, AIM POINT = 0			
σ_3/σ_1	A.R. PROBABILITY	M.A.R. PROBABILITY	M.A.R. AIM POINT
0	0.954	0.954	0
0.2	0.953	0.953	0
0.4	0.950	0.950	0
0.6	0.948	0.948	0
0.8	0.941	0.941	0
1.0	0.934	0.934	0
1.2	0.927	0.927	0
1.4	0.919	0.919	0
1.6	0.910	0.910	0
1.8	0.905	0.905	0
2.0	0.895	0.895	0
2.2	0.890	0.890	0
2.4	0.884	0.884	0
2.6	0.878	0.878	0
2.8	0.875	0.875	0
3.0	0.870	0.870	0
3.2	0.868	0.868	0
3.4	0.866	0.866	0
3.6	0.862	0.862	0
3.8	0.860	0.860	0
4.0	0.857	0.857	0
4.2	0.855	0.855	0
4.4	0.854	0.854	0
4.6	0.853	0.853	0
4.8	0.851	0.851	0
5.0	0.850	0.850	0

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